

LINEAR ALGEBRA IN A NUTSHELL

A is $n \times n$

Nonsingular

A is invertible
The columns are independent
The rows are independent
The determinant is not zero
 $Ax = 0$ has one solution $x = 0$
 $Ax = b$ has one solution
 A has n (nonzero) pivots
 A has full rank $r = n$
The reduced row echelon form is $R = I$
The column space is all of R^n
The row space is all of R^n
All eigenvalues are nonzero
 $A^T A$ is symmetric positive definite
 A has n (positive) singular values

Singular

A is not invertible
The columns are dependent
The rows are dependent
The determinant is zero
 $Ax = 0$ has infinitely many solutions
 $Ax = b$ has no solution or infinitely many
 A has $r < n$ pivots
 A has rank $r < n$
 R has at least one zero row
The column space has dimension $r < n$
The row space has dimension $r < n$
Zero is an eigenvalue of A
 $A^T A$ is only semidefinite
 A has $r < n$ singular values

from Introduction to Linear Algebra (Gilbert Strang) Third Edition, 2003